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The Comparison of Bondage Number and the Average Distance of an Interval Graph

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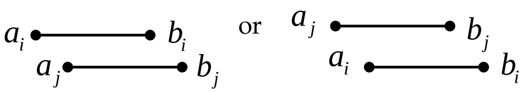
ABSTRACT- Interval graphs have drawn the attention of many researchers for over 30 years. They are extensively studied and revealed their practical relevance for modeling problems arising in the real world. In an interval graph G = (V,E) the distance between two vertices u, v is defined as the smallest number of edges in a path joining u and v. The aim of this paper is to show that the comparison of bondage number and the average distance of an interval graph corresponding to an interval family I.

KEYWORDS - Average Distance, Bondage Number, Distance, Domination Number, Dominating Set, Interval graph.

1 INTRODUCTION

The family of intervals is called an interval family of the graph. Let I={I_1 ,I_2 ,…….I_n } be an interval family where each I_i is an interval on the real line and I_i =[a_i,b_i], for i=1,2,3,……n. Hear a_i is called the left end point and b_i the right end I_i . Without loss of generality, we may assume that all end points of the intervals in I are distinct numbers between 1 and 2n. Two intervals I_i and I_j are said to intersect each other if they have non-empty intersection. That is, if I_i =[a_i,b_i], I_j =[a_j,b_j], then I_i and I_j intersect, means either $a_j < b_i$ or $a_i < b_j$. an interval family I is said to be proper if no interval in I_i is contained in other interval. A graph G (V,E) is called an interval graph if there is a one to one correspondence between V and I such that two vertices of G are joined by an edge E if and only if their corresponding intervals in I intersect. That is if I_i =[a_i,b_i] and I_j =[a_j,b_j], then I_i and I_j intersect means either $a_j < b_i$ or $a_i < b_j$.



A subset DS of V is said to be a dominating set of G if every vertex not in DS is adjacent to vertex in DS. The domination number γ(G) of a graph G is the minimum cardinality of a dominating set in G[5,6]. The bondage number b(G) of a non empty graph G is the minimum cardinality among all sets of edges E_1 for which $\gamma(G-E_1) > \gamma(G)$[2].

The distance between two vertices u and v of a graph is the length of the shortest path. The set of all central vertices of a graph is called the centre of the graph G. The diameter of a graph G is the eccentricity of all its vertices[1].

$$\mathrm{Diam}(G) = \max\{e(v);\ v \in V(G)\}$$

The maximum distance from a vertex u to any vertex of g is called eccentricity of the vertex v.

$$\mathrm{Ece}\ (G) = \max\{d(u,v);\ v \in V(G)\}$$

The radius of a graph G is the minimum of eccentricity of all its vertices.

$$\mathrm{Rad}\ (G) = \min\{e(v);\ v \in V(G)\}$$

In this section we discuss about the computation of average distance of an interval graph G. The average distance μ(G) of a connected interval graph is defined to be the average of all distances in G[7,8,9].

$$\mu(G) = \frac{1}{n(n-1)} \sum_{\substack{x,y \in v(G) \\ x \neq y}} \delta(x,y)$$

$$\text{or} \quad \mu(G) = \frac{1}{2\ {}^{n}c_2} \sum_{\substack{x,y \in v(G) \\ x \neq y}} \delta(x,y)$$

Where δ(x,y) denotes the length of shortest path joining the vertices x and y .

The average distance can be used as a tool in analytic networks where the performance time is proportions to the distance between any two nodes. It is a measure of the time needed in the average case as opposed to the diameter, which indicates the maximum performance time. And also the formulated of the walk, length of a walk, eccentricity, radius and diameter of the graph.

A walk from v_0 to v_n is an alternating sequence W= $\{v_0, e_1, v_1, e_2, \ldots v_{n-1}, e_n, v_n\}$ of vertices and edges such that end points $(e_i)=\{v_{i-1}, v_i\}$; for i={1,2,…n}.

In a simple graph, there is only one edge between two consecutive vertices of a walk, so one could abbreviate the walk as W=$\{v_0, e_1, v_1, e_2, \ldots v_{n-1}, e_n, v_n\}$.

The length of a walk or directed walk is the number of edge steps in the walk sequence. A walk of length zero. i.e with one vertex and no edges is called a trivial walk.

2. MAIN THEOREMS

Theorem 1.

Let G be an interval graph corresponding to the interval family I= $\{a_i, a_j, a_k, \ldots\ldots a_n\}$. Let $(a_i,a_j) \in$ I and suppose a_j is contained in $a_i \neq 1$ and there is no other interval that intersect j, other than i, then the bondage number b(G) is greater than or equal to the average distance of μ(G). where

$$\mu(G) = \frac{1}{2\ {}^{n}c_2} \sum_{\substack{a_i,a_j \in v(G) \\ a_i \neq a_j}} \delta(a_i, a_j).$$

Proof:

Let I= $\{a_i, a_j, a_k, \ldots\ldots a_n\}$ be an interval family and let G be the interval graph corresponding to the given interval family I. Let (a_i,a_j) be any two intervals in I, which satisfy the hypothesis of the theorem. Then clearly $a_i \in DS$, where DS is the minimum dominating set of an interval graph G. Because there is no other interval in I, other than a_i, that dominates a_j consider the edge e = (a_i,a_j) in G.

If we remove this edge from an interval graph G, then a_j becomes an isolated vertex in an interval graph G-e, as there is no other vertex in G, other than a_i, i.e adjacent with a_j.

Hence DS_1 = DS ∪ $\{a_j\}$ becomes a dominating set of an interval graph G-e and since DS is a minimum dominating set of G it follows that DS_1 is also a minimum dominating set of an interval graph G-e.

Therefore we get $|DS_1| = \gamma(G\text{-}e) = |DS|+1 > |DS_1|$

Thus the bondage number b(G)=1. And again we will show that the average distance of an interval graph $\mu(G) = \frac{1}{2\ {}^{n}c_2} \sum_{\substack{a_i,a_j \in v(G) \\ a_i \neq a_j}} \delta(a_i, a_j)$.

In this connection first we will discuss the distance an interval graph G.

For any two vertices a_i,a_j in an interval graph G, the distance from a_i to a_j is denoted by $d(a_i,a_j)$ and defined as the length of a shortest a_i-a_j path in an interval graph G.

The term distance we just defined satisfies all four of the following axioms.

1.d $(a_i,a_j) \geq 0$, for all $a_i,a_j \in V(G)$.

2.d $(a_i,a_j) = 0$, if and only if $a_i = a_j$.

3.d $(a_i,a_j) = d(a_j,a_i)$, for all $a_i,a_j \in V(G)$.

4.d $(a_i,a_k) \leq d(a_i,a_j) + d(a_j,a_k)$, for all $a_i,a_j,a_k \in V(G)$.

If graph G is not a connected and suppose G_1 and G_2 are two components of G, is denoted by W(G).

Which are

$W(G)=W(G_1) \cup W(G_2)$ and $E(W(G_1)) \cup E(W(G_2))$

And $W(G_1) \cap W(G_2) = \emptyset$

That is $V(W(G_1)) \cap V(W(G_2)) = \emptyset$ and also

$E(W(G_1)) \cap V(W(G_2)) = \emptyset$

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Then d $(a_i,a_j)=\infty$ for $a_i \in V(W(G_1))$ & $V \in V(W(G_1))$

Under this distance function the set V(G) is a metric space.

In this fact the interval graph corresponding to interval family I= $\{a_i,a_j,a_k,\ldots\ldots\ldots a_n\}$, where n is a number of vertices.

In this section we discuss about the computation of average distance of an interval graph G. The average distance $\mu(G)$ of a connected an interval graph is defined to be the average of distance in G.

Where $$\mu(G)=\frac{1}{2\ {}^{n}c_2}\sum_{\substack{a_i,a_j\in v(G)\\ a_i\neq a_j}}\delta(a_i,a_j).$$

Therefore the theorem is hold.

2.1 ILLUSTRATION

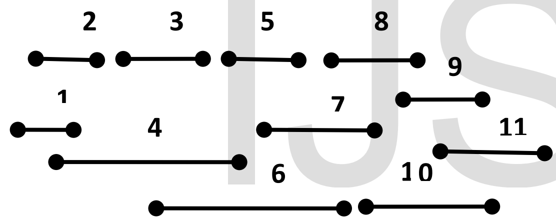

Fig.1: Interval family I

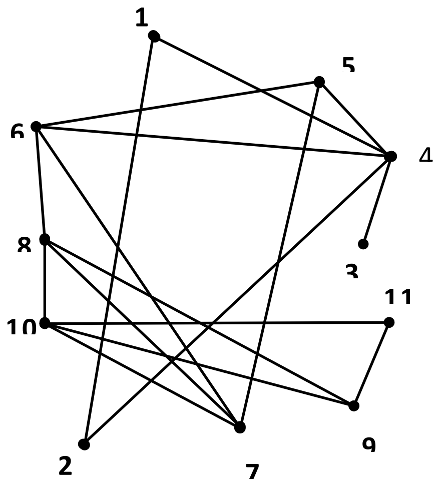

Fig. 2: Interval graph G

Dominating Set = {4,10}, γ(G)=2,

Remove the one edge e= (7,10),

G-e = {G-(7,10)}.

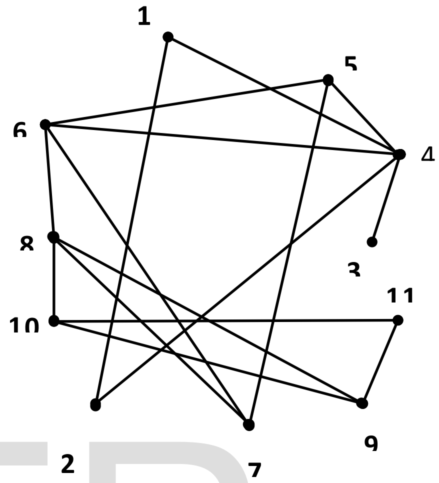

Fig. 3: Interval graph (G-e)

DS of {G-e} = {G-(7,10)} = {4,7,10},

γ(G-e) = 3, also

DS of {G-e} = {G-(1,4)} = {1,4,10},

γ(G-e) = 3

DS of {G-e} = {G-(4,6)} = {4,6,10},

γ(G-e) = 3

Therefore γ(G-e) > γ(G), b(G) = 1

2.2 To find the distances from G

d(1,1)=0	d(2,1)=1	d(3,1)=2	d(4,1)=1
d(1,2)=1	d(2,2)=0	d(3,2)=2	d(4,2)=1
d(1,3)=2	d(2,3)=2	d(3,3)=0	d(4,3)=1
d(1,4)=1	d(2,4)=1	d(3,4)=1	d(4,4)=0
d(1,5)=2	d(2,5)=2	d(3,5)=2	d(4,5)=1
d(1,6)=2	d(2,6)=2	d(3,6)=2	d(4,6)=1
d(1,7)=3	d(2,7)=3	d(3,7)=3	d(4,7)=2
d(1,8)=3	d(2,8)=3	d(3,8)=3	d(4,8)=2
d(1,9)=4	d(2,9)=4	d(3,9)=4	d(4,9)=3
d(1,10)=4	d(2,10)=4	d(3,10)=4	d(4,10)=3

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1	0	1	2	1	2	2	3	3	4	4	5
2	1	0	2	1	2	2	3	3	4	4	5
3	2	2	0	1	2	2	3	3	4	4	5
4	1	1	1	0	1	1	2	2	3	3	4
5	2	2	2	1	0	1	1	2	3	2	3
6	2	2	2	1	1	0	1	1	2	2	3
7	3	3	3	2	1	1	0	1	2	1	2
8	3	3	3	2	2	1	1	0	1	1	2
9	4	4	4	3	2	2	1	1	0	1	2
10	4	4	4	3	2	2	1	1	1	0	1
11	5	5	5	4	3	3	2	2	2	1	0
Total	27	27	28	19	18	17	16	19	26	23	32

Table-1

d(5,1)=2	d(6,1)=2	d(7,1)=3	d(8,1)=3
d(5,2)=2	d(6,2)=2	d(7,2)=3	d(8,2)=3
d(5,3)=2	d(6,3)=2	d(7,3)=3	d(8,3)=3
d(5,4)=1	d(6,4)=1	d(7,4)=2	d(8,4)=2
d(5,5)=0	d(6,5)=1	d(7,5)=1	d(8,5)=2
d(5,6)=1	d(6,6)=0	d(7,6)=1	d(8,6)=1
d(5,7)=1	d(6,7)=1	d(7,7)=0	d(8,7)=1
d(5,8)=2	d(6,8)=1	d(7,8)=1	d(8,8)=0
d(5,9)=3	d(6,9)=2	d(7,9)=2	d(8,9)=1
d(5,10)=2	d(6,10)=2	d(7,10)=1	d(8,10)=1
d(5,11)=3	d(6,11)=3	d(7,11)=2	d(8,11)=2

d(9,1)=4	d(10,1)=4	d(11,1)=5
d(9,2)=4	d(10,2)=4	d(11,2)=5
d(9,3)=4	d(10,3)=4	d(11,3)=5
d(9,4)=3	d(10,4)=3	d(11,4)=4
d(9,5)=2	d(10,5)=2	d(11,5)=3
d(9,6)=2	d(10,6)=2	d(11,6)=3
d(9,7)=1	d(10,7)=1	d(11,7)=2
d(9,8)=1	d(10,8)=1	d(11,8)=2
d(9,9)=0	d(10,9)=1	d(11,9)=2
d(9,10)=1	d(10,10)=0	d(11,10)=1
d(9,11)=2	d(10,11)=1	d(11,11)=0

2.4 To find the average distance of G

vertices	1	2	3	4	5	6	7	8	9	10	11

Therefore distance

$\mu(G) = -$).

$\mu(G) = -$

$\mu(G) =$

d(11,1)=5
d(11,2)=5
d(11,3)=5
d(11,4)=4
d(11,5)=3
d(11,6)=3
d(11,7)=2
d(11,8)=2

2.3 To find the dis

d(1,1)=0	d(2,1)=1	d(3,1)=2	d(4,1)=1
d(1,2)=1	d(2,2)=0	d(3,2)=2	d(4,2)=1
d(1,3)=2	d(2,3)=2	d(3,3)=0	d(4,3)=1
d(1,4)=1	d(2,4)=1	d(3,4)=1	d(4,4)=0
d(1,5)=2	d(2,5)=2	d(3,5)=2	d(4,5)=1
d(1,6)=2	d(2,6)=2	d(3,6)=2	d(4,6)=1
d(1,7)=3	d(2,7)=3	d(3,7)=3	d(4,7)=2
d(1,8)=3	d(2,8)=3	d(3,8)=3	d(4,8)=2
d(1,9)=4	d(2,9)=4	d(3,9)=4	d(4,9)=3
d(1,10)=4	d(2,10)=4	d(3,10)=4	d(4,10)=3
d(1,11)=5	d(2,11)=5	d(3,11)=5	d(4,11)=4

d(5,1)=2 d(6,1)=2 d(7,1)=3 d(8,1)=3
d(5,2)=2 d(6,2)=2 d(7,2)=3 d(8,2)=3
d(5,3)=2 d(6,3)=2 d(7,3)=3 d(8,3)=3
d(5,4)=1 d(6,4)=1 d(7,4)=2 d(8,4)=2
d(5,5)=0 d(6,5)=1 d(7,5)=1 d(8,5)=2
d(5,6)=1 d(6,6)=0 d(7,6)=1 d(8,6)=1
d(5,7)=1 d(6,7)=1 d(7,7)=0 d(8,7)=1

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d(9,1)=4	d(10,1)=4	d(11,1)=5
d(9,2)=4	d(10,2)=4	d(11,2)=5
d(9,3)=4	d(10,3)=4	d(11,3)=5
d(9,4)=3	d(10,4)=3	d(11,4)=4
d(9,5)=3	d(10,5)=3	d(11,5)=4
d(9,6)=2	d(10,6)=2	d(11,6)=3
d(9,7)=2	d(10,7)=2	d(11,7)=3
d(9,8)=1	d(10,8)=1	d(11,8)=2
d(9,9)=0	d(10,9)=1	d(11,9)=1
d(9,10)=1	d(10,10)=(	d(11,10)=1
d(9,11)=1	d(10,11)=:	d(11,11)=0

2.5 To find the Average distance from G-e

V	1	2	3	4	5	6	7	8	9	10	11
1	0	1	2	1	2	2	3	3	4	4	5
2	1	0	2	1	2	2	3	3	4	4	5
3	2	2	0	1	2	2	3	3	4	4	5
4	1	1	1	0	1	1	2	2	3	3	4
5	2	2	2	1	0	1	1	2	3	3	4
6	2	2	2	1	1	0	1	1	2	2	3
7	3	3	3	2	1	1	0	1	2	2	3
8	3	3	3	2	2	1	1	0	1	1	2
9	4	4	4	3	3	2	2	1	0	1	1
10	4	4	4	3	3	2	2	1	1	0	1
11	5	5	5	4	4	3	3	2	1	1	0
Total	27	27	28	19	21	17	21	19	25	25	33

Table-2

Therefore Average distance

$$\mu(G-e)=\frac{1}{2\ {}^{n}c_{2}}\sum_{\substack{a_i,a_j\in v(G-e)\\ a_i\neq a_j}}\delta(a_i,a_j).$$

$$\mu(G-e)=\frac{1}{10\times 11}\times 262$$

$$\mu(G-e)=2.38$$

3. ILLUSTRATION

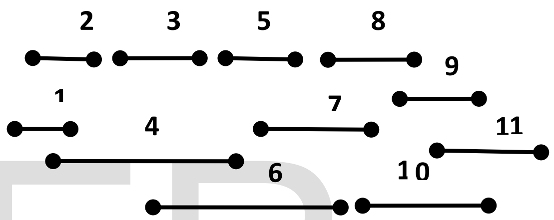

Fig.4 : Interval family I

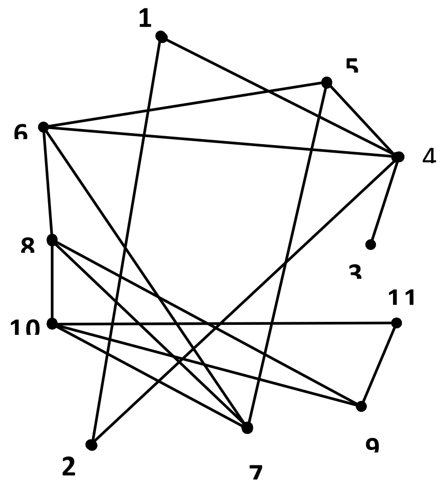

Fig. 5 : Interval graph G

Where G = G-e. Then the corresponding graph is

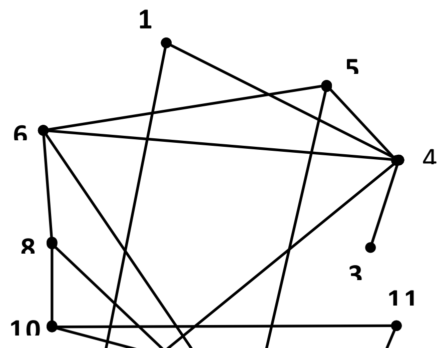

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Fig. 6 : Interval graph (G-e)

Remove the one edge e = (1,4)

Then the corresponding graph is

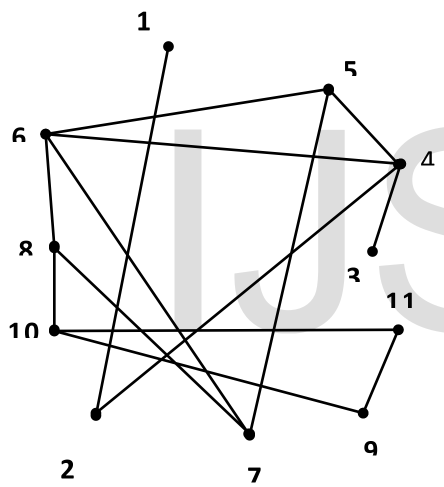

Fig. 7 : Interval graph G^1 ={(G-e)-e}

d(5,1)=3	d(6,1)=3	d(7,1)=4	d(8,1)=4
d(5,2)=2	d(6,2)=2	d(7,2)=3	d(8,2)=3
d(5,3)=2	d(6,3)=2	d(7,3)=3	d(8,3)=3
d(5,4)=1	d(6,4)=1	d(7,4)=2	d(8,4)=2
d(5,5)=0	d(6,5)=1	d(7,5)=1	d(8,5)=2
d(5,6)=1	d(6,6)=0	d(7,6)=1	d(8,6)=1
d(5,7)=1	d(6,7)=1	d(7,7)=0	d(8,7)=1
d(5,8)=2	d(6,8)=1	d(7,8)=1	d(8,8)=0
d(5,9)=3	d(6,9)=2	d(7,9)=2	d(8,9)=1
d(5,10)=3	d(6,10)=2	d(7,10)=2	d(8,10)=1
d(5,11)=4	d(6,11)=3	d(7,11)=3	d(8,11)=2

Dominating set of G^1 ={1,4,8,11}, $\gamma(G^1) = 4$

Therefore $\gamma(G^1) > \gamma(G\text{-}e)$

Therefore B(G) = 1

3.1 To find the distances from G^1

d(1,1)=0	d(2,1)=1	d(3,1)=3	d(4,1)=2
d(1,2)=1	d(2,2)=0	d(3,2)=2	d(4,2)=1
d(1,3)=3	d(2,3)=2	d(3,3)=0	d(4,3)=1
d(1,4)=2	d(2,4)=1	d(3,4)=1	d(4,4)=0
d(1,5)=3	d(2,5)=2	d(3,5)=2	d(4,5)=1
d(1,6)=3	d(2,6)=2	d(3,6)=2	d(4,6)=1
d(1,7)=4	d(2,7)=3	d(3,7)=3	d(4,7)=2
d(1,8)=4	d(2,8)=3	d(3,8)=3	d(4,8)=2

d(9,1)=5	d(10,1)=5	d(11,1)=6
d(9,2)=4	d(10,2)=4	d(11,2)=5
d(9,3)=4	d(10,3)=4	d(11,3)=5
d(9,4)=3	d(10,4)=3	d(11,4)=4
d(9,5)=3	d(10,5)=3	d(11,5)=4
d(9,6)=2	d(10,6)=2	d(11,6)=3
d(9,7)=2	d(10,7)=2	d(11,7)=3
d(9,8)=1	d(10,8)=1	d(11,8)=2
d(9,9)=0	d(10,9)=1	d(11,9)=1
d(9,10)=1	d(10,10)=0	d(11,10)=1
d(9,11)=1	d(10,11)=1	d(11,11)=0

3.2 To find the Average distance from G^1

V	1	2	3	4	5	6	7	8	9	10	11
1	0	1	3	2	3	3	4	4	5	5	6
2	1	0	2	1	2	2	3	3	4	4	5
3	3	2	0	1	2	2	3	3	4	4	5
4	2	1	1	0	1	1	2	2	3	3	4
5	3	2	2	1	0	1	1	2	3	3	4
6	3	2	2	1	1	0	1	1	2	2	3
7	4	3	3	2	1	1	0	1	2	2	3
8	4	3	3	2	2	1	1	0	1	1	2
9	5	4	4	3	3	2	2	1	0	1	1
10	5	4	4	3	3	2	2	1	1	0	1
11	6	5	5	4	4	3	3	2	1	1	0
Total	**36**	**27**	**29**	**20**	**22**	**18**	**22**	**20**	**26**	**26**	**34**

Table-3

The Average distance of

$$G^1 = \mu(G^1) = \frac{1}{2\ {}^{n}c_2} \sum_{\substack{a_i, a_j \in v(G^1) \\ a_i \neq a_j}} \delta(a_i, a_j).$$

$$\mu(G^1) = \frac{1}{11 \times 10} \times 280$$

$$\mu(G^1) = 2.545$$

$$\therefore \ \mu(G^1) > \gamma(G)$$

Correspondence theorem of G^1 from G

Theorem: 2

Let G^1 = G-e is an interval graph corresponding to an interval family I. Let (a_i,a_j) Є I and suppose a_j is contained in a_i, $a_i \neq 1$ and there is no other interval that intersects a_j other than a_i. Then the average distance of G^1 strictly greater than the bondage number $b(G^1)$.

Proof:

Let G^1 = G-e be the interval graph corresponding to the given interval family I. Let (a_i,a_j) be any two intervals in I, which satisfy the hypothesis of the theorem .

Then clearly $a_i \in DS$, where DS is a minimum dominating set of G^1 because there is no other interval in I, other than a_i , that dominates a_j.

Consider the edge e = (a_i,a_j) in G^1. If we remove this edge from G^1, then a_j becomes an isolated vertex in G^1-e , there is no other vertex in G^1 ,other than a_i , that intersect with a_j.

Hence $DS_1 = DS \cup \{a_j\}$ becomes a dominating set of G^1-e and since DS is a minimum dominating set of G^1.

It follows that DS_1 is also a minimum dominating set of G^1-e.

Therefore $|DS_1| = \gamma(G^1\text{-e}) = |DS| + 1 > |DS_1|$

Thus the bondage number $b(G^1) = 1$ and

Therefore the cardinality of bondage number $|b(G^1)|=1$

Next we will show that the average distance strictly greater than the bondage number of $b(G^1)$ we consider the interval family $I=\{a_i,a_j,a_k,\ldots\ldots.a_n\}$ and let G^1 be interval graph corresponding to an interval family I.

For two vertices a_i, a_j in a graph G^1, the distance from a_i to a_j is denoted by $d(a_i,a_j)$ and defined as the length of a shortest a_i,-a_j path in graph G^1 and already we proved this process in theorem.1.

The Average distance $\mu(G^1)$ of a connected an interval graph is defined to be the average of all distances in G^1.

Where $\mu(G^1)=\dfrac{1}{n(n-1)}\sum\limits_{\substack{a_i,a_j\in v(G^1)\\ a_i\neq a_j}}\delta(a_i,a_j)$

or

$$\mu(G^1)=\frac{1}{\frac{2n(n-1)}{2}}\sum_{\substack{a_i,a_j\in v(G^1)\\ a_i\neq a_j}}\delta(a_i,a_j)$$

or

$$\mu(G^1)=\frac{1}{2\,{}^nC_2}\sum_{\substack{a_i,a_j\in v(G^1)\\ a_i\neq a_j}}\delta(a_i,a_j)$$

And therefore the average between the G^1 its greater than or equal to 2. …….. (1)

Since the bondage number $b(G) = 1$………….. (2)

From (1) and (2) we get $\mu(G^1)>b(G^1)$

Or

$\dfrac{1}{2\,{}^nC_2}\sum\limits_{\substack{a_i,a_j\in v(G^1)\\ a_i\neq a_j}}\delta(a_i,a_j)$ is greater than b(G-e)

Theorem 3:

Let G and G^1 be an interval graphs correspond to the interval family I. Let $a_{i,}a_j\in I$ and suppose a_j is contained in contained in a_i. $a_i\neq 1$ and there is no other interval that intersects a_j, other than a_i, then the average distance of G^1 is greater than the average distance of G.

Proof:

This proof is already proved in the theorem of G and the theorem of G1 that is $\mu(G^1)>\mu(G)$ and also we write of G and G1 corresponding to and interval family I.

$I=\{a_i,a_j,a_k,\ldots\ldots\ldots\ldots\ldots a_n\}$ that is

$$\frac{1}{2\,{}^nC_2}\sum_{\substack{a_i,a_j\in v(G^1)\\ a_i\neq a_j}}\delta(a_i,a_j)>\frac{1}{2\,{}^nC_2}\sum_{\substack{a_i,a_j\in v(G)\\ a_i\neq a_j}}\delta(a_i,a_j)$$

Therefore the average distance of G^1 strictly is greater than the average distance of G.

CONCLUSION

In this paper we study the comparison of bondage number and the average distance of an interval graph corresponding to an Interval family I. In future, efforts in the paper eventually open up many an avenue in the field of research on interval graph.

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